Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

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ABSTRACT

In the paper vacuum pressure is found from the space geometry around the local gravity system according to Sakharov's idea of a "metrical elasticity" of space. It is assumed that energy equivalent to the gravitational defect of masses is spent on vacuum deformation. We determined the gravitational impact on the vacuum in the case of weakly gravitating static centrally symmetric distribution of matter using appropriate solution of Einstein's equations. This impact is balanced by the vacuum pressure of the opposite sign. The equation of state includes the density of matter and the vacuum pressure. They correspond to the field equations for centrally symmetric distribution of matter. The photon motion in the resulting space-time is considered and the energy-momentum-mass vector corresponding to the (1 + 4) D Extended Space Model (ESM) is found. We study abilities to apply the equation of state obtained to arbitrary We study abilities to apply the equation and momentum in Friedmann-Lemaitre-Robertson-Walker space-time is described using rotation angles in 5D. This cosmological model yields about half dark energy density not associated with cosmological constant with half dark energy density not associated with cosmological constant with half dark energy density not associated with cosmologic deceleration parameter $q_0 = -0.425$ at present epoch.

Keywords: Cosmology; extended space model; gravitational impact; non-zero vacuum pressure.

1 INTRODUCTION

Non-zero vacuum pressure is an element in cosmological models [1-3] arising from solutions of the Einstein equations. Einstein postulated that the curvature of space-time is responsible for gravity. Sakharov argued [4] that gravity arises from quantum field theory in much the same sense as hydrodynamics or continuum elasticity arises from molecular physics. Sakharov suggested that the curvature of space "..leads to the "metric elasticity" of space, i.e., to generalized forces that counteract the curvature of space." The action in Einstein's geometrodynamics was identified with changes in the quantum fluctuations of the vacuum [5-8]. It follows from the general relativity that more compactly located matter distorts space to a greater extent in a local area and creates a smaller gravitational mass compared to the same amount of matter distributed over a larger volume [1,2]. In [9], this phenomenon is explained by the appearance of negative binding energy in the gravitational system. The accumulation of energy during deformation demonstrates the elasticity of space. We will take these properties of gravity into account when determining the vacuum pressure. The resulting equation of state is investigated for agreement with cosmological models.

The problem of combining the electromagnetic and gravitational fields into one single field has been discussed since the end of the nineteenth century. It is characteristic that all these attempts were made on the way of constructing geometric models of physical interactions and interpreting physics as

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New Trends in Physical Science Research Vol. 6 Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

geometry in spaces of a larger number of dimensions. F. Klein [10] proposed to consider the Hamilton-Jacobi theory as an optics in the space of the highest number of dimensions. A new surge of interest to this problem was stimulated by the creation of the general theory of relativity. T. Kaluza [11] extended it to five dimensions. O. Klein [12] gave its five-dimensional theory a quantum interpretation. Subsequently, various versions of 5D theory based on the Kaluza-Klein approach were proposed [13-16]. One variant of them is the space-time-matter theory or induced matter theory [14,15], in which the role of 4D uncharged mass is played in 5D geometry by the extra coordinate.

ESM [17,18] is a special theory of relativity generalization in the 5-dimensional space G(1,4) having an additional coordinate *S*, which is an extension of the action concept of the embedded 4D space [19,20] in 5D space. As a parameter determining the movement of a particle in an additional dimension, the optical refractive index of the medium is taken, which sets the speed of movement of the photon in the coordinate reference frame [21]. Lorentz transformations in the (1,3) Minkowski space are complemented in (1,4) ESM by rotations in planes (TS) and (XS) [18]. Rotations of the energy-momentum-mass vector in the extended space corresponds to the gravity field particle motion in the embedded four-dimensional space-time [22,23]. Movement along additional 5-th coordinate in ESM is associated with existing particles variable rest-mass processes consideration, for example electron-positron annihilation. We obtain the energy-momentum-mass 5-vector of a photon in homogeneous sphere by means of sequence of rotations (TS) and (XS). A relation between these angles of rotation in 5D space-time with the universe expansion deceleration parameter is considered.

2. SOLUTION OF EINSTEIN EQUATIONS FOR SPHERICAL SOURCE

We analyse gravitational field with centrally symmetrical source, by selecting units of measurement, in which a light velocity *c*, a gravitational constant *G* and Planck constant *h* are $c = 8\pi G = h = 1$. This gravitational field is described [24] in spherical coordinates $x^i = (t, r, \theta, \varphi)$ by the metric

$$ds^{2} = e^{\nu(r,t)}dt^{2} - e^{\omega(r,t)}dr^{2} - r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}),$$
(1)

where v, ω are the functions of time and radial coordinate. For this metric Einstein equations are

$$e^{-\omega} \left(\frac{1}{r}\omega' - \frac{1}{r^2}\right) + \frac{1}{r^2} = T_1^1, \qquad (2)$$

$$-e^{-\omega}\left(\frac{1}{r}v'+\frac{1}{r^2}\right) + \frac{1}{r^2} = T_2^2, \qquad (3)$$
$$-\frac{1}{r}e^{\omega}\left(v''+\frac{1}{r}(v)^2 + \frac{1}{r}(v'-w)\right) - \frac{1}{r}v'(v') + \frac{1}{r^2}e^{-\omega}(v''-w) + \frac{1}{r^2}e^{-\omega}(v$$

$$\frac{1}{2}e^{-\nu}\left(\ddot{\omega} + \frac{1}{2}(\dot{\omega})^2 - \frac{1}{2}\dot{\nu}\dot{\omega}\right) = T_3^3 = T_4^4,$$
(4)

$$-e^{-\omega}\frac{1}{r}\dot{\omega} = T_1^2,\tag{5}$$

$$e^{-\nu}\frac{1}{r}\dot{\omega} = T_2^1,\tag{6}$$

where () and () denote the derivatives with respect to r and t respectively, T_j^i is a stress-energy tensor.

Solving equation (1) with respect to ω gives the function

$$\omega = -\ln\left(1 - \frac{1}{r}\int_0^r T_1^1 y^2 dy\right).$$
(7)

Subtracting equation (3) from (2) we find

$$\frac{e^{-\omega}}{r}(\omega'+\nu') = (T_1^1 - T_2^2).$$
(8)

Substituting eq. (7) into (8) and solving it with respect to $\boldsymbol{\nu}$ we find

Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

$$\nu = \ln\left(1 - \frac{1}{r} \int_0^r T_1^1 y^2 dy\right) + \int \frac{(T_1^1 - T_2^2)r}{1 - \frac{1}{r} \int_0^r T_1^1 y^2 dy} dr + A,$$
(9)

where *A* is a constant. A Schwarzschild solution $v = -\omega$ is appropriate for the spherical source with radius *a* in the outer area $r \ge a$. In the inner area of the sphere the required function [25] is

$$\nu = \ln\left(1 - \frac{1}{r}\int_0^r T_1^1 y^2 dy\right) - \int_r^a \frac{(T_1^1 - T_2^2)z}{1 - \frac{1}{z}\int_0^z T_1^1 y^2 dy} dz.$$
(10)

3. PRESSURE IN LOW GRAVITATING SPHERE

The static spherical source of gravitation with constant density is considered. The stress-energy tensor of matter is

$$T_j^i = (\rho + p)u^i u_j - \delta_j^i p, \tag{11}$$

where ρ is density, p is pressure, u^i is four-velocity vector and δ^i_j is the Kronecker delta. The first component of this tensor in considered case reduces to

$$T_1^1 = \rho. \tag{12}$$

In the external area the functions v, w (7), (10) correspond to the Schwarzschild metric. Therefore, the value

$$M = 4\pi \int_0^a \rho r^2 dr \tag{13}$$

is the gravitational mass of a spherical body with radius *a*. Integration is performed here by the element of space volume $dV_c = 4\pi r^2 dr$, that corresponds to coordinate frame. The proper volume element, built on the elements of spatial coordinates, is defined [1] as follows:

$$dV_p = \sqrt{\det\left[\frac{g_{1q}g_{1l}}{g_{11}} - g_{ql}\right]}$$
(14)

with indexes q, l = 2,3,4. For the considered volume element of the spacetime, described by (1), this yields $dV_p = 4\pi r^2 e^{\omega/2} dr$. From (7) the inequality $\omega > 0$ means that the gravitational mass of the body is less than the sum of the individual gravitational masses of the constituent elements.

The volume of the spherical body in the proper frame is obtained by integration of the element dV_p with (7) and amounts to

$$V_{int}^{p}(a) = \int_{0}^{a} 4\pi r^{2} e^{\omega/2} dr = \int_{0}^{a} 4\pi r^{2} \left(1 - \frac{1}{3}\rho r^{2}\right)^{-1/2} dr.$$
(15)

The expression under integral is expanded into a formal power series for small space curvature inside the sphere, i.e. with $\rho a^2 \ll 1$, this integral gives

$$V_{int}^{p}(a) = \frac{4\pi}{3}a^{3} + \frac{2\pi}{15}\rho a^{5}.$$
 (16)

The mass of the body in this frame or the proper mass will be $M^p = \rho V_{int}^p(a)$. A proper energy of static source of gravitation is defined as $E^p = M^p$.

The gravitational impact on the vacuum is determined as the relation between the difference of the proper energies of two spherical bodies with identical gravitational mass and the difference of the proper space volumes of the two bodies. With densities ρ_1, ρ_2 and radii $a_1, a_2, (a_1 < a_2)$ this mass is

Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

$$M = \frac{4}{3}\pi\rho_1 a_1^3 = \frac{4}{3}\pi\rho_2 a_2^3.$$
 (17)

The difference of proper masses of two bodies is written as follows:

$$\Delta M^p = M_1^p - M_2^p = \frac{2\pi}{15} a_1^6 \rho_1^2 \left(\frac{1}{a_1} - \frac{1}{a_2}\right).$$
(18)

Both bodies curve the space equally in the area $r > a_2$ due to the equality of their gravitational masses. Let's find the difference between the volumes in the proper frame, that are set in coordinate frame by the condition $r \le a_2$. The volume of the first body is the sum of the proper volume and the volume of the peripheral region $a_1 < r \le a_2$, namely,

$$V_1^p = V_{\text{int}}^p(a_1) + V_{\text{ext}}^p(a_1, a_2),$$
(19)

where the second term is given by

$$V_{ext}^{p}(a_{1},a_{2}) = \int_{a_{1}}^{a_{2}} 4\pi r^{2} e^{\omega/2} dr.$$
 (20)

Function ω in the outer region takes the form

$$\omega_{ext} = -\ln\left(1 - \frac{M}{r}\right) \tag{21}$$

and breaking the expression under integral into the formal power series in case of $M/r \ll 1$ we obtain

$$V_{ext}^{p}(a_1, a_2) = \frac{4}{3}\pi(a_2^3 - a_1^3) + 2\pi M(a_2^2 - a_1^2).$$
⁽²²⁾

As a result, the volume (19) will amount to

$$V_1^p = \frac{4}{3}\pi a_2^3 + \frac{1}{15}\pi \rho_1 a_1^3 (5a_2^2 - 3a_1^2).$$
⁽²³⁾

The area $r \le a_2$ restricts the second body, whose proper volume for the weak gravitational field according to (16) is

$$V_2^p = \frac{4}{3}\pi a_2^3 + \frac{2}{15}\pi \rho_2 a_2^5.$$
⁽²⁴⁾

The difference between the proper volumes, confined within the radius a_2 in coordinate frame, will be

$$\Delta V^p = V_1^p - V_2^p = \frac{1}{5}\pi \rho_1 a_1^3 (a_2^2 - a_1^2).$$
⁽²⁵⁾

With the conservation of the gravitational mass of the spherical body (17), the ratio of change in its proper energy $\Delta E^p = \Delta M^p$ (18) to the change of the volume for small $\Delta a = a_2 - a_1$ yields

$$\wp = \frac{\Delta E^p}{\Delta V^p} = \frac{1}{3}\rho. \tag{26}$$

In the theory of elasticity \wp corresponds to the pressure of an perfect liquid. Positive pressure of gravity field characterizes the gravitational impact of matter on the vacuum, which lies in its constraint. This relationship between density and pressure coincides with the state equation of an ideal relativistic gas [1] and a photon gas [24].

The gravity field pressure upon vacuum is compensated by pressure of the vacuum itself:

Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

 $p_v = -\wp. \tag{27}$

This is mean vacuum pressure in case of weak gravitation inside the static sphere. It is assumed that the vacuum pressure that enters into the energymomentum tensor is the source of the gravitational field.

4. SOLUTION FOR VACUUM PRESSURE $p = -(1/3)\rho$

We find solutions for the field equations (2)-(6) for the gravitational system that includes a spherical source of gravity with a constant density and corresponding vacuum pressure (27). Derivatives of functions (7) and (10) with respect to r are

$$\omega' = -e^{\omega} \left(\frac{1}{r^2} \int_0^r T_1^1 y^2 dy - T_1^1 r \right),$$
(28)

$$\nu' = e^{\omega} \left(\frac{1}{r^2} \int_0^r T_1^1 y^2 dy - T_2^2 r \right).$$
⁽²⁹⁾

The components of the energy-momentum tensor (11) are reduced to

$$T_2^2 = T_3^3 = T_4^4 = -p = \frac{1}{3}\rho.$$
(30)

Together with component (12) this yields

$$\omega' = \frac{2}{3}e^{\omega}r\rho,\tag{31}$$

$$\nu = 0. \tag{32}$$

Substituting these values in equations (2)-(6) gives the identity. Inside the sphere due to equation (10) coefficient ν will be

$$\nu = \ln\left(1 - \frac{1}{3}\rho a^2\right). \tag{33}$$

In the inner region with ω (7) corresponding to component of the stress-energy tensor (12) metric (1) takes the form

$$ds^{2} = \left(1 - \frac{1}{3}\rho a^{2}\right)dt^{2} - \frac{dr^{2}}{1 - \frac{1}{3}\rho r^{2}} - r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}).$$
(34)

The time dilation will be the same inside the sphere and at the surface. This metric can be used to determine the course of time in the interior of the Earth.

We consider the motion of a particle described by the space-time (34). The energy-momentum vector of a free moving photon π^i is obtained by principle of light-like particle extremal energy integral [22,25-27]. It gives generalized momenta

$$p_i = \frac{u_i}{u^1 u_1},\tag{35}$$

which are constant for cyclic coordinates. In plane $\theta = \pi/2$ these constants are

$$p_1 = \frac{1}{u^1}, p_4 = \frac{u_4}{u^1 u_1}.$$
(36)

They are chosen so that for r = a the energy-momentum vector of a photon corresponds to the value in the external Schwarzschild field

Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

$$\pi_{s}^{i} = \nu_{0} \left(\frac{1}{1 - \frac{\alpha}{a}}, \pm \sqrt{1 - \frac{B^{2}}{a^{2}} \left(1 - \frac{\alpha}{a} \right)}, 0, \frac{B}{a^{2}} \right),$$
(37)

where v_0 is the photon frequency at infinity, *B* and $\alpha = 2GM$ are constants. From (34) it follows

$$\left(1 - \frac{\alpha}{a}\right)(u^{1})^{2} - \frac{(u^{2})^{2}}{1 - \frac{1}{3}\rho r^{2}} - r^{2}[(u^{3})^{2} + \sin^{2}\theta(u^{4})^{2}] = 0$$
(38)

and the 4-velocity vector of a photon inside the sphere becomes

$$u^{i} = \left(1, \pm \sqrt{\left(1 - \frac{\alpha}{a} - \frac{A^{2}}{r^{2}}\right)\left(1 - \frac{1}{3}\rho r^{2}\right)}\right), 0, \frac{A}{r^{2}}\right)$$
(39)

for $A = -p_4 \left(1 - \frac{\alpha}{a}\right)$. The energy-momentum vector of a photon $\pi^i = v_0 p^i$ is

$$\pi^{i} = \nu_{0} \left(1 - \frac{\alpha}{a} \right)^{-1} \left(1, \pm \sqrt{\left(1 - \frac{\alpha}{a} - \frac{A^{2}}{r^{2}} \right) \left(1 - \frac{1}{3} \rho r^{2} \right)} \right), 0, \frac{A}{a^{2}} \right).$$
(40)

5. ISOTROPIC SOURCE

The ability to apply the equation of state defined by (30) to arbitrary gravitational systems is studied. Let us examine a space-time described by the metric $ds^2 = g_{ij}dx^i dx^j$ and containing a source of gravitation with density ρ . It is assumed that in a small area, whose boundary is a sphere in the proper frame, the metrical coefficients and density can be considered as constants and the pressure isotropic. The gravity, created by this ball, is described by metric (1).

The metrical coefficients of the space-time without a source of gravitation in this sphere will be slightly different from g_{ij} . The transition to a locally inertial system [1] with the beginning in the point x_0^k is made for the changed metrics using the transformation

$$x^{\prime k} = x^{k} + \frac{1}{2} \left(\Gamma_{ij}^{k} \right)_{x^{i} = x_{0}^{i}} x^{i} x^{j}$$
(41)

with Christoffel's symbols Γ_{ij}^k . In this locally flat space we place the absent source of gravitation in the empty sphere. This is comparable to the conditions under which the pressure of the gravitational field was obtained (26). The proper pressure of vacuum is determined for static space-time according to (27) and will be

$$p_{\nu} = -\frac{1}{3}\rho. \tag{42}$$

6. TRANSFORMATION OF ENERGY AND MOMENTUM IN ESM

In accordance with ESM, we find the energy-momentum-mass 5-vector of a free moving photon in homogeneous sphere defined by metric (34) using rotations (TS) and (XS). Each particle energy-momentum 4-vector is completed in extended space G(1,4) [17,28] with an additional coordinate *s* to 5-vector

$$\bar{p} = (E, p_x, p_y, p_z, m),$$
 (43)

where *m* is a rest mass of the particle. In empty space this vector corresponds to two types of objects with zero and nonzero masses respectively in a fixed reference system. For simplicity we will record it in (1 + 2)-dimensional space as

$$\bar{p} = (E, P, p_s). \tag{44}$$

Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

Its hyperbolic rotations in the planes (TS) and (XS) yields

$E' = E \mathrm{cosh} \varphi_{TS} + p_s \mathrm{sinh} \varphi_{TS}$,	(45)
P'=P,	(46)
$p_{s}^{'}=E{ m sinh}arphi_{TS}+p_{s}{ m cosh}arphi_{TS}$,	(47)

and

$$E' = E,$$

$$P' = P \cosh \varphi_{XS} + p_s \sinh \varphi_{XS},$$

$$p'_s = P \sinh \varphi_{XS} + p_s \cosh \varphi_{XS},$$
(49)
(49)
(50)

where φ_{TS} and φ_{XS} are angles of rotations.

A photon localization in ESM [27] means that it gets a non-zero mass entering into the external field. The corresponding infinite plane wave is compressed to a finite size in according to de Broglie formula. We consider a sphere with a mirror inner surface filled with uniformly distributed photons, which will correspond to metric (34). Formally, we can extend the geodesic of a photon beyond the sphere, where it corresponds to the Schwarzschild metric. The 5D energymomentum-mass vector of a photon

$$\bar{p} = (E, P, 0) \tag{51}$$

can be transformed into a vector with components equal to the energy and momentum in Schwarzschild space-time at low gravity using (TS) rotation [22]. This rotation changes the energy of a photon, leaving momentum in 4D constant. According to (40), the energy of a photon inside the sphere remains constant while momentum changes. Hence, when it enters the sphere, the change in the vector of the energy-momentum of a photon is described by (XS) transformation. The sequence of rotations (TS)-(XS) of the vector (51) gives

$$E^{''} = E \cosh \varphi_{TS}, \tag{52}$$

$$P^{''} = E \sinh \varphi_{TS} + P \cosh \varphi_{TS} \tag{53}$$

$$P = E \operatorname{sinn} \varphi_{XS} \operatorname{sinn} \varphi_{TS} + P \operatorname{cosn} \varphi_{XS}, \tag{53}$$

$$p_s = E \cosh \varphi_{XS} \sinh \varphi_{TS} + P \sinh \varphi_{XS}. \tag{54}$$

Comparing the resulting vector (40) agreeing with equation (52), we find the (TS) rotation angle for the radial motion

$$\varphi_{TS} = \cosh^{-1} \left[\left(1 - \frac{\alpha}{a} \right)^{-1} \right].$$
(55)

Equation (53) takes the form

$$\pm \sqrt{\frac{\left(1 - \frac{1}{3}\rho r^2\right)}{\left(1 - \frac{\alpha}{a}\right)}} = \cosh\varphi_{XS} + \sqrt{\left(\cosh^2\varphi_{XS} - 1\right)\left(\frac{2\alpha}{a} - \frac{\alpha^2}{a^2}\right)} \left(1 - \frac{\alpha}{a}\right)^{-1}$$
(56)

and has solution

$$\varphi_{XS} = \cosh^{-1} \left[\left(\pm \sqrt{\left(1 - \frac{1}{3}\rho r^2\right)} \left(1 - \frac{\alpha}{a}\right)^{3/2} \sqrt{\left(\frac{2\alpha}{a} - \frac{\alpha^2}{a^2}\right) \left(\frac{3\alpha}{a} - \frac{2\alpha^2}{a^2} - \left(1 - \frac{\alpha}{a}\right)\frac{1}{3}\rho r^2\right)} \right) \left(1 - \frac{4\alpha}{a} + \frac{2\alpha^2}{a^2}\right)^{-1} \right].$$
(57)

The obtained rotation angles allow us to determine the momentum $p_s^{"}$ (54), which, according to the ESM, is associated with the mass of the particle.

The (TS) rotation corresponds to a transition from the density of the static matter to the density as a source of gravity [19,30]. The energy distribution of a massive particle in the form of a Gauss function [29] can be considered as a possible reason for the appearance of vacuum pressure in the area of matter distribution.

7. COSMOLOGICAL APPLICATION

In FLRW cosmological model the source of gravitation is static in comoving coordinates, which is locally geodesic system. In rectangular coordinates $x^i = (t, x^q)$ it is described by metric

$$ds^{2} = dt^{2} - \frac{a^{2}(t)dx^{q^{2}}}{(1+K[(x^{2})^{2}+(x^{3})^{2}+(x^{4})^{2}])^{2}}$$
(58)

with expansion factor *a* and spatial curvature *K*. Equation of state (42) corresponds to static or uniformly expanding space-time. The FLRW model with a(t) = t [31,32] meets this condition. If expansion of the universe is accelerating, the inequality $-p_v > \wp$ holds. However, the relative velocity of the expansion of the Universe is equal to Hubble parameter *H*, which is small at present period, and this equation of state is suitable.

The photon energy and momentum corresponding to a space-time (58) at K = 0 [33] are

$$E = v_0 a^{-1}, \quad P = v_0 a^{-2}. \tag{59}$$

According to (45) and (49), they can be obtained by rotating in 5D the vector (51) at the angles

$$\varphi_{TS} = \cosh^{-1}(a^{-1}) \tag{60}$$

and

$$\varphi_{XS} = \cosh^{-1}(a^{-2}). \tag{61}$$

The expansion of the Universe is characterized by the deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2},\tag{62}$$

which is 0 for a uniformly expanding universe. The deceleration parameter can be expressed in terms of rotation angles (60), (61) as

$$q = 1 - (csch\varphi_{TS})^2 - \frac{\dot{\varphi}_{TS}}{\dot{\varphi}_{TS}^2} coth\varphi_{TS}$$
(63)

and

$$q = \frac{(csch\varphi_{XS})^{1/2}}{4} \bigg[1 - 3(csch\varphi_{XS})^2 - 2\frac{\ddot{\varphi}_{XS}}{\dot{\varphi}_{XS}^2} coth\varphi_{XS} \bigg].$$
(64)

The angles φ_{TS} , φ_{XS} are small in the present cosmological epoch and these expressions reduce to

$$q_0 = -\frac{\dot{\varphi}_{TS}}{\dot{\varphi}_{TS}^2 \varphi_{TS}} \tag{65}$$

and

$$q_0 = -\frac{1}{2} \left(1 + \frac{\ddot{\varphi}_{XS}}{\dot{\varphi}_{XS}^2 \varphi_{XS}} \right). \tag{66}$$

Estimates of the deceleration parameter are based on type Ia supernovae observations analysis. It is stated in [34,35] that these observations provide evidence of the accelerated expansion of the universe at present cosmological epoch, which corresponds to the deceleration parameter $q_0 =$

Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

 $-0.425^{+0.116}_{-0.122}$ [34]. This value of q_0 is permissible in cosmological model [30] taking into account pressure of vacuum excited by matter (42).

We assume that dark energy consists of two components, one of which corresponds to the cosmological constant Λ . Its density parameter is

$$\Omega_{DE} = \Omega_{\Lambda} + \Omega_{DE'} \tag{67}$$

where $\Omega_{DE'}$ is the density of dark energy not associated with Λ . In Λ CDM cosmology the deceleration parameter is represented in form

$$q = \frac{1}{2} \sum \Omega_i (1 + 3w_i), \tag{68}$$

where Ω_i are the density parameters for different components: matter (*m*), radiation (*r*), dark energy (Λ, DE') , spatial curvature (*K*) and $w_i = p_i/\rho_i$ is the equation of state for each component. We consider a model, in which coefficient w_i is written in form

$$w_i = w_i^p + w_i^e, ag{69}$$

where $w_i^p = p_i^p / \rho_i$ corresponds the component's own pressure and $w_i^e = p_i^e / \rho_i$ appears due to the excited pressure of vacuum.

From the Friedmann equations it follows $w_{\Lambda}^p = -1$. Ellis [36,37] didn't introduce the pressure of a substance into the energy-momentum tensor, which we agree with: $w_m^p = w_{DE'}^p = 0$. It is replaced by pressure of a vacuum $w_m^e = w_{DE'}^e = w_{\Lambda}^e = -1/3$. We consider values Ω_r at present time and Ω_K according to the Planck 2018 results [37] to be negligible. As a result, the deceleration parameter (68) is

$$q = -\frac{3}{2}\Omega_{\Lambda} \,. \tag{70}$$

For $q = q_0$ relation (70) yields the density of associated with Λ dark energy at present time $\Omega_{\Lambda} = 0.283^{+0.077}_{-0.081}$. Constraints on the total density of dark energy in Λ CDM cosmology [38] give positive value $\Omega_{DE'} \gtrsim 0.5\Omega_{DE}$.

8. CONCLUSION

Matter warps space, and it is natural to assume that the result is a vacuum pressure. We have examined a possible mechanism for the occurrence of this pressure, based on the gravitational defect of the masses and the assumption of elasticity of space in accordance with the law of energy conservation. The spherical sources of gravity with constant densities and identical gravitational masses is considered in the spheres with the same volume in the remote frame. The difference between the proper volume of the spheres and their volume in the remote frame increases with the increase in mass defect, which gives a positive pressure of gravitational field. In statics the vacuum pressure balances impact of gravity on vacuum according to the theory of elasticity. The resulting vacuum pressure corresponds to the solution of Einstein's equations, in which the course of time inside the sphere is constant. Obtained metric can be used to determine the course of time in the interior of the Earth. In ESM the 5D energy-momentum-mass vector of a photon in flat space can be transformed into a vector with components equal to its energy and momentum inside the sphere by sequence of rotations (TS)-(XS).

We have shown that equation of state, obtained for a weakly gravitating sphere, can be extended to a distributed locally isotropic static gravity source. It excites vacuum acquiring pressure corresponding to w = -(1/3). Mechanism of excitation of vacuum pressure by matter and cosmological constant was applied to the FLRW cosmology. This results in a model, in which only the cosmological constant

Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

determines the dynamics of the expansion of the Universe and its energy density is close to the half of the dark energy density.

The energy and momentum of a photon in FLRW space-time can be expressed through rotations of the energy-momentum-mass 5 -vector. This allows us to determine the deceleration parameter as a function of the rotation angles.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Homogeneous Sphere with Excited Vacuum Pressure, Applications in Extended Space Model and Cosmology

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